OCR
Oxford Cambridge and RSA

Monday 25 June 2018 – Morning

A2 GCE MATHEMATICS (MEI)

4756/01 Further Methods for Advanced Mathematics (FP2)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4756/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.
- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** If additional space is required, you should use the lined page(s) at the end of this booklet. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

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- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR
- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (54 marks)

1 **(a)** The polar equation of a curve is $r = a\sin^2\theta\cos\theta$ for $0 \leqslant \theta \leqslant \frac{1}{2}\pi$.

(i) Find the value of θ for which the curve has the maximum x-coordinate. **[3]**

(ii) Prove that the maximum y-coordinate on the curve is $\frac{3\sqrt{3}}{16}a$ and state the value of θ at which this is attained. **[4]**

(b) **(i)** Sketch the graph of $y = \arcsin x$ for $-1 \leqslant x \leqslant 1$. **[1]**

(ii) Prove that $\frac{\mathrm{d}}{\mathrm{d}x}(\arcsin x) = \frac{1}{\sqrt{1-x^2}}$. **[4]**

(iii) Using integration by parts and a suitable substitution, show that

$$\int_0^1 x^2 \arcsin x \, \mathrm{d}x = \frac{3\pi - 4}{18}.$$ **[6]**

2 **(a)** **(i)** Use de Moivre's theorem to prove that

$$\cot 4\theta = \frac{1 - 6\tan^2\theta + \tan^4\theta}{4\tan\theta(1-\tan^2\theta)}.$$ **[5]**

(ii) Hence express the roots of the equation

$$x^4 + 4x^3 - 6x^2 - 4x + 1 = 0$$

in exact trigonometrical form. **[4]**

(b) The vertices of a square with sides of length 1 unit lie on the axes of an Argand diagram. The vertices represent the complex numbers z_1, z_2, z_3 and z_4 and the midpoints of the sides of the square represent the complex numbers z_5, z_6, z_7 and z_8.

(i) Express z_5, z_6, z_7 and z_8 in modulus-argument form, and hence determine a polynomial equation of degree 4, with integer coefficients, whose roots are z_5, z_6, z_7 and z_8. **[4]**

Let $\mathrm{P}(z) = 0$ be a polynomial equation of degree 8, with integer coefficients, whose roots are z_1, z_2, z_3, z_4, z_5, z_6, z_7 and z_8.

(ii) Explain why $\mathrm{P}(z)$ cannot be of the form $az^8 + b$ where a and b are integers. **[1]**

(iii) Find $\mathrm{P}(z)$. **[4]**

3 **(i)** Find the inverse of the matrix $\begin{pmatrix} 1 & 3 & 2 \\ -1 & 2 & k \\ k & 1 & 6 \end{pmatrix}$. **[5]**

The matrix **M** has eigenvalues 1, 2 and 3. The corresponding eigenvectors are $\begin{pmatrix} 2 \\ 0 \\ 6 \end{pmatrix}$, $\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$ respectively.

(ii) Write down the matrix **P** such that $\mathbf{M} = \mathbf{PDP}^{-1}$ where $\mathbf{D} = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$. **[2]**

(iii) Hence find **M**. **[5]**

(iv) Find constants a, b and c such that $\mathbf{M}^{-1} = a\mathbf{M}^2 + b\mathbf{M} + c\mathbf{I}$. **[6]**

Section B (18 marks)

4 **(i)** Prove, using definitions in terms of exponential functions, that

$$\cosh 2A = 1 + 2\sinh^2 A.$$ **[3]**

(ii) Find $\int \sinh^2 x \, dx$. **[3]**

(iii) Let $z = \text{arsinh}(1)$. Form an equation involving z and solve it to find the exact value of arsinh(1) in logarithmic form. **[4]**

(iv) Using a substitution of the form $ax = b\sinh u$, find the exact value of

$$\int_0^{\frac{2}{3}} \frac{x^2}{\sqrt{4+9x^2}} \, dx,$$

giving your answer in the form $p(q - \ln r)$, where p, q and r are constants. **[8]**

END OF QUESTION PAPER

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4756/01 Further Methods for Advanced Mathematics (FP2)

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Duration: 1 hour 30 minutes

Candidate forename		Candidate surname	

Centre number						Candidate number				

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Section A (54 marks)

1 (a)(i)	
1 (a)(ii)	

1 (b)(i)	
1 (b)(ii)	
1 (b)(iii)	
	(answer space continued on next page)

(answer space continued on next page)

1 (b)(iii)	(continued)

2 (a)(i)

2 (a)(ii)

2 (b)(i)	
2 (b)(ii)	
2 (b)(iii)	

3 (i)	
3 (ii)	

3 (iii)	
3 (iv)	

Section B (18 marks)

4 (i)	
4 (ii)	
4 (iii)	

4 (iv)

ADDITIONAL ANSWER SPACE

If additional space is required, you should use the following lined page(s). The question number(s) must be clearly shown in the margin(s).

OCR
Oxford Cambridge and RSA

GCE

Mathematics (MEI)

Unit **4756**: Further Methods for Advanced Mathematics

Advanced GCE

Mark Scheme for June 2018

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, Cambridge Nationals, Cambridge Technicals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support, which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Annotations and abbreviations

Annotation in scoris	**Meaning**
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

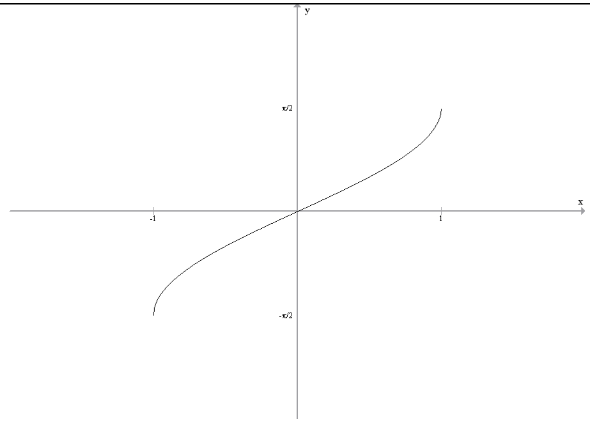
If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance
1	(a)	(i)	$x = r\cos\theta = a\sin^2\theta\cos^2\theta$ $= \frac{a}{4}\sin^2 2\theta$ which is maximum when $2\theta = \frac{\pi}{2}$ i.e. $\theta = \frac{\pi}{4}$ *(Accept 0.785)*	M1 DM1 A1 **3**	Expressing x in terms of θ For $x = k\sin^2 2\theta$ **or** using $\frac{dx}{d\theta} = 0$ to obtain an equation for θ [$2a(\sin\theta\cos^3\theta - \sin^3\theta\cos\theta) = 0$] cwo Can only be given if M1M1 are earned *Alternative* **M2** for a complete method using cartesian equation leading (via $y^2 = x^2$) to a value for θ
1	(a)	(ii)	$y = r\sin\theta = a\sin^3\theta\cos\theta$ $\frac{dy}{d\theta} = 3a\sin^2\theta\cos^2\theta - a\sin^4\theta = 0$ $\Rightarrow \sin\theta = 0$ (reject) or $\tan\theta = \sqrt{3}$ (rej. –ve) $\Rightarrow \theta = \frac{\pi}{3}$ *(Accept 1.05)* $y_{max} = \frac{a}{2}.\left(\frac{\sqrt{3}}{2}\right)^3 = \frac{3\sqrt{3}}{16}a$	M1 A1 A1 A1 **4**	Finding $dy/d\theta$ **or** complete method using Cartesian equation leading to a value for θ or y_{max} Correct equation for θ (**or** $y^2 = 3x^2$) *Dependent on M1A1; can be verified* **AG**. cwo Intermediate step required (e.g. $a\sin^3\left(\frac{\pi}{3}\right)\cos\frac{\pi}{3}$)
1	(b)	(i)		B1 **1**	Correct curvature, through origin, and values at end points made clear. *Accept 90* *Ignore anything drawn outside -1<x<1* *Lenient on gradient at O and endpoints* *Must be single-valued (lenient)* *Accept interchanged axes if correctly labelled*

Question			Answer	Marks	Guidance
1	(b)	(ii)	let $\sin y = x$ $\frac{dy}{dx} = \frac{1}{\cos y}$ $\frac{dy}{dx} = (\pm)\frac{1}{\sqrt{1-\sin^2 y}}$ $= (\pm)\frac{1}{\sqrt{1-x^2}}$ but $\frac{d(\arcsin x)}{dx} > 0$ so $\frac{d(\arcsin x)}{dx} = \frac{1}{\sqrt{1-x^2}}$	M1 DM1 A1 A1 **4**	Differentiating $\sin y = x$ Using $\cos^2 y = 1 - \sin^2 y$ cwo **or** $\cos y > 0$ since $-\frac{\pi}{2} < y < \frac{\pi}{2}$, (so $\cos y = +\sqrt{1-\sin^2 y}$) **AG** *Dependent on all previous marks*
1	(b)	(iii)	$\int_0^1 x^2 \arcsin x dx = \left[\frac{x^3}{3}\arcsin x\right]_0^1 - \int_0^1 \frac{x^3}{3}.\frac{1}{\sqrt{1-x^2}} dx$ Let $J = \int_0^1 \frac{x^3}{3}.\frac{1}{\sqrt{1-x^2}} dx$ Let $u = \sqrt{1-x^2}$, $du = -x(1-x^2)^{-1/2} dx$ (or $u^2 = 1 - x^2$, $2u du = -2x dx$) So $J = -\frac{1}{3}\int(1-u^2) du$ $J = -\frac{1}{3}\left[u - \frac{u^3}{3}\right]$ $\int_0^1 x^2 \arcsin x \, dx = \left(\frac{1}{3}.\frac{\pi}{2} - 0\right) + \frac{1}{3}\left(0 - 1 + \frac{1}{3}\right)$ $\left(= \frac{\pi}{6} - \frac{2}{9}\right) = \frac{3\pi - 4}{18}$	B1 M1 A1 DM1 DM1 A1 **6**	Ignore limits **or** $u = 1 - x^2$, $du = -2x dx$ **or** $x = \sin\theta$, $dx = \cos\theta \, d\theta$ $J = -\frac{1}{6}\int\frac{(1-u)}{\sqrt{u}} du$ $J = \frac{1}{3}\int \sin^3\theta d\theta = \frac{1}{3}\int(\sin\theta - \cos^2\theta\sin\theta)d\theta$ *or* $\frac{1}{3}\int(\frac{3}{4}\sin\theta - \frac{1}{4}\sin 3\theta)d\theta$ $J = -\frac{1}{6}\left[2u^{1/2} - \frac{2}{3}u^{3/2}\right]$ $J = \frac{1}{3}\left[-\cos\theta + \frac{1}{3}\cos^3\theta\right]$ Applying limits correctly to both parts **AG** Everything must be fully correct *Alternative for first two marks* **M1** for $x = \sin\theta$, $dx = \cos\theta \, d\theta$ applied to original integral **B1** for $\int \theta\sin^2\theta\cos\theta d\theta = \frac{1}{3}\theta\sin^3\theta - \int\frac{1}{3}\sin^3\theta d\theta$

Question			Answer	Marks	Guidance
2	(a)	(i)	$(\cos\theta+\mathrm{j}\sin\theta)^4=\cos^4\theta+4\mathrm{j}\cos^3\theta\sin\theta$ $-6\cos^2\theta\sin^2\theta-4\mathrm{j}\cos\theta\sin^3\theta+\sin^4\theta$ $\cos4\theta=\cos^4\theta-6\cos^2\theta\sin^2\theta+\sin^4\theta$ $\sin4\theta=4\cos^3\theta\sin\theta-4\cos\theta\sin^3\theta$ $\cot4\theta=\frac{\cos^4\theta-6\cos^2\theta\sin^2\theta+\sin^4\theta}{4\cos^3\theta\sin\theta-4\cos\theta\sin^3\theta}$ and dividing num. and denom. by $\cos^4\theta$ gives $\cot4\theta=\frac{1-6\tan^2\theta+\tan^4\theta}{4\tan\theta(1-\tan^2\theta)}$ as req'd.	M1 A1 A1 DM1 A1 **5**	Expanding $(\cos\theta+\mathrm{j}\sin\theta)^4$ Forming cot4θ **AG**. Must indicate division by $\cos^4\theta$ *(M0 for methods other than deMoivre)*
2	(a)	(ii)	Given quartic eqn is formed from $\cot4\theta=1$ (with $x=\tan\theta$) i.e. $\tan4\theta=1\Rightarrow4\theta=\frac{\pi}{4},\frac{5\pi}{4},\frac{9\pi}{4},\frac{13\pi}{4}$ first solution is $\theta=\frac{\pi}{16}$ $x=\tan\frac{\pi}{16}$ $x=\tan\left(\frac{5\pi}{16}\right),\tan\left(\frac{9\pi}{16}\right),\tan\left(\frac{13\pi}{16}\right)$	B1 B1 B1 ft B1 **4**	Any one value of θ *(allow* $\frac{1}{4}\arctan1$ *)* Any one root (ft requires $\theta\neq n\pi/2$) *Ignore repeats; B0 if any incorrect (Accept any exact form)*

Question			Answer	Marks	Guidance
2	(b)	(i)	Midpoints are $\frac{1}{2}e^{\pm\frac{1}{4}\pi j}$, $\frac{1}{2}e^{\pm\frac{3}{4}\pi j}$ eqn is of the form $z^4 = k$ subst. gives $k = -\frac{1}{16}$ i.e. $16z^4 + 1 = 0$	B1 B1 M1 A1 **4**	For modulus ½ For arguments $\pm\frac{1}{4}\pi$, $\pm\frac{3}{4}\pi$ (o.e.) *(allow, e.g.* $\frac{1}{2}\left(\cos\frac{\pi}{4} - \mathrm{j}\sin\frac{\pi}{4}\right)$ *)* Must have integer coefficients
2	(b)	(ii)	P(z) cannot be of the form $az^8 + b$ as the vertices and midpoints of the sides do not form a regular octagon.	B1 **1**	**Or** equivalent algebraic consideration based upon the two quartic equations. **Or** roots do not all have the same modulus
2	(b)	(iii)	Vertices of original square are $\pm\frac{1}{\sqrt{2}}$ and $\pm\frac{1}{\sqrt{2}}\mathrm{j}$ Equation satisfied by these is $4z^4 - 1 = 0$ Eqn having midpts and vertices as roots is $(4z^4 - 1)(16z^4 + 1) = 0$ $\mathrm{P}(z) = 64z^8 - 12z^4 - 1$	B1 M1 A1 A1 **4**	For $\left(z^4 - \frac{1}{4}\right)$ or $z^4 = \frac{1}{4}$ For $(z^4 - k)(\text{polynomial from (i) of degree 4})$ For $(z^4 - k)\left(z^4 + \frac{1}{4}k\right)$ for any $k > 0$ [e.g. $(z^4 - 1)(4z^4 + 1)$] (implied by $z^8 - mz^4 - \frac{4}{9}m^2$ or $\frac{4}{9}m^2z^8 - mz^4 - 1$) [e.g. $4z^8 - 3z^4 - 1$] *(Allow factorised form)* Must have integer coefficients

Question		Answer	Marks	Guidance
3	(i)	Let **X** = $\begin{pmatrix} 1 & 3 & 2 \\ -1 & 2 & k \\ k & 1 & 6 \end{pmatrix}$		
		det **X** = $3k^2 - 5k + 28$	M1	Allow one error
		$\mathbf{X}^{-1} = \frac{1}{3k^2 - 5k + 28}\begin{pmatrix} 12-k & -16 & 3k-4 \\ k^2+6 & 6-2k & -2-k \\ -1-2k & 3k-1 & 5 \end{pmatrix}$	M1 A1 DM1 A1	At least 4 (signed) cofactors correct. **M0** if multiplied by the corresponding element. 6 (signed) cofactors correct. Transposing and multiplying by 1/det **X** cao.
			5	
3	(ii)	**P** = $\begin{pmatrix} 1 & 3 & 2 \\ -1 & 2 & 0 \\ 0 & 1 & 6 \end{pmatrix}$	M1 A1	Using the three eigenvectors as columns of a 3x3 matrix. (**M1A0** if columns are in the wrong order)
			2	
3	(iii)	Using result from **(i)**, with k = 0, $\mathbf{P}^{-1} = \frac{1}{28}\begin{pmatrix} 12 & -16 & -4 \\ 6 & 6 & -2 \\ -1 & -1 & 5 \end{pmatrix}$	M1	Or from scratch with fewer than 3 errors
		M = **PDP**$^{-1}$		
		PD = $\begin{pmatrix} 3 & 6 & 2 \\ -3 & 4 & 0 \\ 0 & 2 & 6 \end{pmatrix}$	M1 A1 ft	First product. **NB** **DP**$^{-1} = \frac{1}{28}\begin{pmatrix} 36 & -48 & -12 \\ 12 & 12 & -4 \\ -1 & -1 & 5 \end{pmatrix}$
		so **M** = $\frac{1}{28}\begin{pmatrix} 70 & -14 & -14 \\ -12 & 72 & 4 \\ 6 & 6 & 26 \end{pmatrix}$	DM1 A1	Other product, giving **M** cao *(Correct answer always earns 5 marks)*
			5	

Question		Answer	Marks	Guidance
3	(iv)	Characteristic equation may be expressed as $(\lambda-1)(\lambda-2)(\lambda-3)=0$	M1	**or** expanding $\det(\mathbf{M}-\lambda\mathbf{I})$ (M0 for $(\lambda+1)$ ...)
		i.e. $\lambda^3-6\lambda^2+11\lambda-6=0$	A1	Alternatively, may be awarded later in terms of **M** and **I** (=0 is not required)
		By the Cayley-Hamilton theorem, **M** must satisfy the characteristic equation, so $\mathbf{M}^3-6\mathbf{M}^2+11\mathbf{M}-6\mathbf{I}=\mathbf{0}$	M1	(=0 required; can be implied later) (**I** not required)
		Multiplying by $\mathbf{M}^{-1}$ gives	M1	Cubic expression needed
		$\mathbf{M}^2-6\mathbf{M}+11\mathbf{I}-6\mathbf{M}^{-1}=\mathbf{0}$	A1 ft	**I** needed (can be recovered later); must be an equation
		$\Rightarrow \mathbf{M}^{-1}=\frac{1}{6}\mathbf{M}^2-\mathbf{M}+\frac{11}{6}\mathbf{I}$	A1	cao $\left(a=\frac{1}{6}, b=-1, c=\frac{11}{6}\right)$
			6	*Alternatively: **M3** for complete method leading to a value for one of a, b, c **A1A1A1** for answers*

Question		Answer	Marks	Guidance
4	(i)	$1+2\sinh^2 A = 1+2\dfrac{(e^A-e^{-A})^2}{4}$ $=1+\dfrac{e^{2A}-2+e^{-2A}}{2}$ $=\dfrac{e^{2A}+e^{-2A}}{2}$ $=\cosh 2A$	M1 A1 A1 **3**	Use of exponential form Use of $(e^A-e^{-A})^2=e^{2A}-2+e^{-2A}$ in correct expression **AG** ***(****Be lenient with questionable logic)*
4	(ii)	$\int \sinh^2 x \; dx = \frac{1}{2}\int(\cosh 2x - 1)\,dx$ $=\frac{1}{4}[\sinh 2x - 2x]+c$	M1 A2,1,0 **3**	**or** $\int \frac{1}{4}(e^{2x}-2+e^{-2x})dx$ **or** $\frac{1}{8}e^{2x}-\frac{1}{2}x-\frac{1}{8}e^{-2x}+c$ -1 each error; $+c$ is required
4	(iii)	$z=\text{arsinh}(1)\Rightarrow \sinh z = 1$ $\Rightarrow \dfrac{e^z-e^{-z}}{2}=1$ $\Rightarrow e^{2z}-2e^z-1=0$ $\Rightarrow (e^z-1)^2=2 \;\Rightarrow\; e^z=1\pm\sqrt{2}$ (but $e^z>0$) $\Rightarrow z=\text{arsinh}(1)=\ln(1+\sqrt{2})$	M1 A1 M1 A1 **4**	Correct quadratic in e^z or, using the formula, $e^z=\frac{2\pm\sqrt{8}}{2}$ *Reason for rejecting* $1-\sqrt{2}$ *not required*

Question		Answer	Marks	Guidance
4	(iv)	$3x = 2\sinh u$	B1	
		$3\mathrm{d}x = 2\cosh u\,\mathrm{d}u$	M1	M0 for e.g. $\frac{du}{dx} = \frac{2}{3}\cosh u$
		$\Rightarrow \int \frac{x^2}{\sqrt{4+9x^2}}\,\mathrm{d}x = \int \frac{(\frac{4}{9}\sinh^2 u)(\frac{2}{3}\cosh u)}{\sqrt{4+4\sinh^2 u}}\,\mathrm{d}u$	DM1	Ignore limits
		$= \frac{4}{27}\int \sinh^2 u\,\mathrm{d}u$	A1	Ignore limits
		$= \frac{1}{27}[\sinh 2u - 2u]$ $= \frac{1}{27}[2\sinh u\cosh u - 2u]$	M1	Using **(ii)**. Ignore limits
			M1	Correct use of limits ($u = \mathrm{arsinh}(1)$ and $u = 0$); or writing in terms of *x* (e.g. $\frac{1}{27}\left[3x\sqrt{1+\frac{9x^2}{4}} - 2\mathrm{arsinh}\left(\frac{3x}{2}\right)\right]$) and using $x = \frac{2}{3}$ and $x = 0$ *Must obtain an exact expression*
		$= \frac{1}{27}\left[2(1)\sqrt{2} - 2\ln\left(1+\sqrt{2}\right)\right]$ $= \frac{2}{27}\left[\sqrt{2} - \ln(1+\sqrt{2})\right]$	A2 **8**	Give A1 for $\lambda\sqrt{2} - \mu\ln(\ldots)$ with $\ln(1+\sqrt{2})$ or answer to (iii); with $\lambda, \mu \neq 0$ and rational; and $\lambda = \frac{2}{27}$ **or** $\mu = \frac{2}{27}$ **or** $\lambda = \mu$ *A2 is dependent on all previous marks* *A1 is dependent on M4*

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Examiners' report

MATHEMATICS (MEI)

3895-3898, 7895-7898

4756/01 Summer 2018 series

Version 1

Contents

Introduction

Our examiners’ reports are produced to offer constructive feedback on candidates’ performance in the examinations. They provide useful guidance for future candidates. The reports will include a general commentary on candidates’ performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report. A full copy of the question paper can be downloaded from OCR.

Paper 4756/01 series overview

Candidates found parts of question 1 (calculus) and question 2 (complex numbers) to be particularly challenging, and the marks on these two questions were generally low. Candidates performed much better on question 3 (matrices) and question 4 (hyperbolic functions).

There are several places where candidates are asked for a proof, or where the answer is given on the examination. In such cases candidates are advised to show every detail of their working, so that they do not lose marks unnecessarily. Examiners will not fill in any gaps in the calculations or the reasoning, however 'obvious' the steps might appear to be. Just copying the given answer will not be given any credit unless it follows from the previous working. Candidates often obtain an answer which differs from the given one; it is then perfectly legitimate to look back, discover the error and make corrections. However, great care is needed, as forgetting to correct a single line could invalidate the whole argument.

Question 1 (a) (i)

1 **(a)** The polar equation of a curve is $r = a\sin^2\theta\cos\theta$ for $0 \leqslant \theta \leqslant \frac{1}{2}\pi$.

(i) Find the value of θ for which the curve has the maximum x-coordinate. **[3]**

The essential first step was to use $x = r\cos\theta$ and the polar equation of the curve to express x in terms of θ. Candidates who did this were very often able to complete this part; some differentiated the expression, and others rewrote it as $x = \frac{1}{4}a\sin^2 2\theta$. Most candidates did not start in the right way, and so earned no credit. Many differentiated the polar equation, as if they were maximising r instead of x. Many obtained the cartesian equation of the curve, but were unable to progress beyond this.

Question 1 (a) (ii)

(ii) Prove that the maximum y-coordinate on the curve is $\frac{3\sqrt{3}}{16}a$ and state the value of θ at which this is attained. **[4]**

To earn any credit in this part it was necessary to write y in terms of θ and obtain $\frac{\mathrm{d}y}{\mathrm{d}\theta}$. Only a small minority of candidates did this. Some tried to use trigonometrical identities to express y in a form where the maximum value could be seen, but this proved to be unsuccessful. Most candidates either omitted this part altogether, or repeated the misunderstandings they had shown in part (i).

Question 1 (b) (i)

(b) **(i)** Sketch the graph of $y = \arcsin x$ for $-1 \leqslant x \leqslant 1$. **[1]**

Most candidates drew a graph with the correct shape. The most common reason for not earning the mark was not indicating the y-coordinates $\left(\pm\frac{1}{2}\pi\right)$ of the end-points.

Question 1 (b) (ii)

(ii) Prove that $\frac{\mathrm{d}}{\mathrm{d}x}(\arcsin x) = \frac{1}{\sqrt{1-x^2}}$. **[4]**

This was very well answered, with most candidates differentiating $\sin y = x$ and then using $\cos^2 y = 1 - \sin^2 y = 1 - x^2$. As the answer is given, a fully detailed explanation is expected, including why the positive square root is taken. Most cited that the gradient of the graph is always positive; and others stated that $\cos y > 0$ because $-\frac{1}{2}\pi < y < \frac{1}{2}\pi$.

Question 1 (b) (iii)

(iii) Using integration by parts and a suitable substitution, show that

$$\int_0^1 x^2 \arcsin x \, dx = \frac{3\pi - 4}{18}. \qquad \textbf{[6]}$$

Almost all candidates applied the formula for integration by parts correctly. Most then selected the substitution $u = 1 - x^2$ or $u^2 = 1 - x^2$ to obtain an integrable form. Many selected $x = \sin u$ but some could not deal with the resulting integral $\int \sin^3 u \, du$. Many solutions were quite difficult to follow, with numerous corrections and changes of sign; and missing d*x*'s and d*u*'s making it uncertain when the change of variables had been completed. Many candidates did not explain clearly how they were dealing with the limits of integration. The answer is given, so to earn full marks the final version presented needs to be accurate and fully explained.

Question 2 (a) (i)

2 **(a)** **(i)** Use de Moivre's theorem to prove that

$$\cot 4\theta = \frac{1 - 6\tan^2\theta + \tan^4\theta}{4\tan\theta(1 - \tan^2\theta)}. \qquad \textbf{[5]}$$

Most candidates understood how to apply deMoivre's theorem to obtain this result. With the answer given, a full explanation was expected, and very many candidates did not state that they were dividing the numerator and denominator by $\cos^4\theta$ to convert the terms into powers of $\tan\theta$. Some candidates ignored the instruction to use deMoivre's theorem and used the formula for $\tan 2\theta$ to obtain the result; this was not given any credit.

Question 2 (a) (ii)

(ii) Hence express the roots of the equation

$$x^4 + 4x^3 - 6x^2 - 4x + 1 = 0$$

in exact trigonometrical form. **[4]**

Most candidates did not see how to relate this equation to part (i), and many omitted this part altogether. Many did use $x = \tan\theta$ to transform the equation to $\cot 4\theta = 1$, but only some of these went on to complete the solution.

Question 2 (b) (i)

(b) The vertices of a square with sides of length 1 unit lie on the axes of an Argand diagram. The vertices represent the complex numbers z_1, z_2, z_3 and z_4 and the midpoints of the sides of the square represent the complex numbers z_5, z_6, z_7 and z_8.

(i) Express z_5, z_6, z_7 and z_8 in modulus-argument form, and hence determine a polynomial equation of degree 4, with integer coefficients, whose roots are z_5, z_6, z_7 and z_8. **[4]**

Most candidates gave the four arguments correctly. Many candidates took the vertices of the square to be at 1, j, –1, –j so that the sides had length $\sqrt{2}$ instead of 1, and therefore gave the wrong modulus for the midpoints. Those who realised that the midpoints represented the fourth roots of a number were able to obtain a polynomial equation; but others tried to expand $(z-z_5)(z-z_6)(z-z_7)(z-z_8)=0$ usually without success.

Question 2 (b) (ii)

Let $\mathrm{P}(z)=0$ be a polynomial equation of degree 8, with integer coefficients, whose roots are z_1, z_2, z_3, z_4, z_5, z_6, z_7 and z_8.

(ii) Explain why $\mathrm{P}(z)$ cannot be of the form az^8+b where a and b are integers. **[1]**

Most candidates who offered a correct explanation stated that the eight complex numbers did not all have the same modulus.

Question 2 (b) (iii)

(iii) Find $\mathrm{P}(z)$. **[4]**

Candidates who recognised that the eight complex numbers consisted of two sets of fourth roots were usually able to make progress.

Question 3 (i)

3 **(i)** Find the inverse of the matrix $\begin{pmatrix} 1 & 3 & 2 \\ -1 & 2 & k \\ k & 1 & 6 \end{pmatrix}$. **[5]**

Most candidates understood how to find the inverse matrix. The only common mistakes were careless errors in the calculation of the determinant or the cofactors.

Question 3 (ii)

The matrix $\mathbf{M}$ has eigenvalues 1, 2 and 3. The corresponding eigenvectors are $\begin{pmatrix} 2 \\ 0 \\ 6 \end{pmatrix}$, $\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$ respectively.

(ii) Write down the matrix $\mathbf{P}$ such that $\mathbf{M}=\mathbf{PDP}^{-1}$ where $\mathbf{D}=\begin{pmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$. **[2]**

Almost all candidates wrote down the correct matrix. Just a few had the columns in the wrong order.

Question 3 (iii)

(iii) Hence find **M**. **[5]**

Most candidates found the inverse of **P** and carried out the matrix multiplications confidently, although careless errors were fairly common.

Question 3 (iv)

(iv) Find constants a, b and c such that $\mathbf{M}^{-1} = a\mathbf{M}^2 + b\mathbf{M} + c\mathbf{I}$. **[6]**

Most candidates approached this by finding the characteristic equation of **M**, applying the Cayley-Hamilton theorem, and then multiplying by $\mathbf{M}^{-1}$. This was very often completed accurately, with the first step being the most likely to go wrong. The characteristic equation was usually obtained correctly when considered as the equation with the eigenvalues (1, 2, 3) as its roots. Many candidates attempted to find it by expanding $\det(\mathbf{M} - \lambda\mathbf{I})$ and this was very rarely successful.

Question 4 (i)

4 **(i)** Prove, using definitions in terms of exponential functions, that

$$\cosh 2A = 1 + 2\sinh^2 A.$$ **[3]**

This was answered very well by most candidates. Some lost marks by missing out steps in the working, for example, writing down $2\sinh^2 A = \frac{1}{2}\left(e^{2A} + e^{-2A} - 2\right)$ without giving the exponential form of $\sinh A$. Some gave the correct working in terms of exponentials, but never related their conclusion to $\cosh 2A$ and $\sinh A$.

Question 4 (ii)

(ii) Find $\int \sinh^2 x \, dx$. **[3]**

This was also answered well, either by using the identity from part (i) or by converting to exponential form. Marks were often lost through careless slips, or by omitting the arbitrary constant.

Question 4 (iii)

(iii) Let $z = \text{arsinh}(1)$. Form an equation involving z and solve it to find the exact value of arsinh(1) in logarithmic form. **[4]**

This was well understood, with almost all candidates obtaining the correct answer.

Question 4 (iv)

(iv) Using a substitution of the form $ax = b\sinh u$, find the exact value of

$$\int_0^{\frac{2}{3}} \frac{x^2}{\sqrt{4+9x^2}}\,dx,$$

giving your answer in the form $p(q - \ln r)$, where p, q and r are constants. **[8]**

Most candidates used the right substitution $3x = 2\sinh u$ and transformed the integral to $\int \frac{4}{27}\sinh^2 u\ du$, although some did not carry out the change of variable correctly. This integral was given by part (ii), and then the most challenging step was to find the exact value of $\sinh 2u$ when. $\sinh u = 1$. Many candidates did this efficiently by writing it as $2\sinh u \cosh u$ or by converting to exponential form.

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AS GCE / Advanced GCE / AS GCE Double Award / Advanced GCE Double Award

AS & Advanced GCE Mathematics											
				Max Mark	a	b	c	d	e	u
4721	01	C1 Core mathematics 1 (AS)	Raw	72	61	55	50	45	40	0
			UMS	100	80	70	60	50	40	0
4722	01	C2 Core mathematics 2 (AS)	Raw	72	55	49	43	37	31	0
			UMS	100	80	70	60	50	40	0
4723	01	C3 Core mathematics 3 (A2)	Raw	72	55	48	41	34	28	0
			UMS	100	80	70	60	50	40	0
4724	01	C4 Core mathematics 4 (A2)	Raw	72	54	47	40	34	28	0
			UMS	100	80	70	60	50	40	0
4725	01	FP1 Further pure mathematics 1 (AS)	Raw	72	56	50	45	40	35	0
			UMS	100	80	70	60	50	40	0
4726	01	FP2 Further pure mathematics 2 (A2)	Raw	72	59	53	47	41	35	0
			UMS	100	80	70	60	50	40	0
4727	01	FP3 Further pure mathematics 3 (A2)	Raw	72	47	41	36	31	26	0
			UMS	100	80	70	60	50	40	0
4728	01	M1 Mechanics 1 (AS)	Raw	72	60	51	42	34	26	0
			UMS	100	80	70	60	50	40	0
4729	01	M2 Mechanics 2 (A2)	Raw	72	53	46	39	32	26	0
			UMS	100	80	70	60	50	40	0
4730	01	M3 Mechanics 3 (A2)	Raw	72	50	42	34	27	20	0
			UMS	100	80	70	60	50	40	0
4731	01	M4 Mechanics 4 (A2)	Raw	72	59	53	47	42	37	0
			UMS	100	80	70	60	50	40	0
4732	01	S1 – Probability and statistics 1 (AS)	Raw	72	57	50	43	36	29	0
			UMS	100	80	70	60	50	40	0
4733	01	S2 – Probability and statistics 2 (A2)	Raw	72	56	49	42	35	28	0
			UMS	100	80	70	60	50	40	0
4734	01	S3 – Probability and statistics 3 (A2)	Raw	72	59	50	41	32	24	0
			UMS	100	80	70	60	50	40	0
4735	01	S4 – Probability and statistics 4 (A2)	Raw	72	56	49	42	35	28	0
			UMS	100	80	70	60	50	40	0
4736	01	D1 – Decision mathematics 1 (AS)	Raw	72	55	48	42	36	30	0
			UMS	100	80	70	60	50	40	0
4737	01	D2 – Decision mathematics 2 (A2)	Raw	72	58	53	48	44	40	0
			UMS	100	80	70	60	50	40	0

AS & Advanced GCE Mathematics (MEI)

				Max Mark	a	b	c	d	e	u
4751	01	C1 – Introduction to advanced mathematics (AS)	Raw	72	60	55	50	45	40	0
			UMS	100	80	70	60	50	40	0
4752	01	C2 – Concepts for advanced mathematics (AS)	Raw	72	53	47	41	36	31	0
			UMS	100	80	70	60	50	40	0
4753	01	(C3) Methods for Advanced Mathematics (A2): Written Paper	Raw	72	61	56	51	46	40	0
4753	02	(C3) Methods for Advanced Mathematics (A2): Coursework	Raw	18	15	13	11	9	8	0
4753	82	(C3) Methods for Advanced Mathematics (A2): Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
			UMS	100	80	70	60	50	40	0
4754	01	C4 – Applications of advanced mathematics (A2)	Raw	90	63	56	49	43	37	0
			UMS	100	80	70	60	50	40	0
4755	01	FP1 – Further concepts for advanced mathematics (AS)	Raw	72	55	51	47	43	40	0
			UMS	100	80	70	60	50	40	0
4756	01	FP2 – Further methods for advanced mathematics (A2)	Raw	72	48	42	36	31	26	0
			UMS	100	80	70	60	50	40	0
4757	01	FP3 – Further applications of advanced mathematics (A2)	Raw	72	63	56	49	42	35	0
			UMS	100	80	70	60	50	40	0
4758	01	(DE) Differential Equations (A2): Written Paper	Raw	72	61	54	48	42	35	0
4758	02	(DE) Differential Equations (A2): Coursework	Raw	18	15	13	11	9	8	0
4758	82	(DE) Differential Equations (A2): Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
			UMS	100	80	70	60	50	40	0
4761	01	M1 – Mechanics 1 (AS)	Raw	72	51	44	37	31	25	0
			UMS	100	80	70	60	50	40	0
4762	01	M2 – Mechanics 2 (A2)	Raw	72	59	53	47	41	35	0
			UMS	100	80	70	60	50	40	0
4763	01	M3 – Mechanics 3 (A2)	Raw	72	61	54	48	42	36	0
			UMS	100	80	70	60	50	40	0
4764	01	M4 – Mechanics 4 (A2)	Raw	72	59	51	44	37	30	0
			UMS	100	80	70	60	50	40	0
4766	01	S1 – Statistics 1 (AS)	Raw	72	59	53	47	42	37	0
			UMS	100	80	70	60	50	40	0
4767	01	S2 – Statistics 2 (A2)	Raw	72	54	47	41	35	29	0
			UMS	100	80	70	60	50	40	0
4768	01	S3 – Statistics 3 (A2)	Raw	72	61	54	47	41	35	0
			UMS	100	80	70	60	50	40	0
4769	01	S4 – Statistics 4 (A2)	Raw	72	56	49	42	35	28	0
			UMS	100	80	70	60	50	40	0
4771	01	D1 – Decision mathematics 1 (AS)	Raw	72	50	44	38	32	26	0
			UMS	100	80	70	60	50	40	0
4772	01	D2 – Decision mathematics 2 (A2)	Raw	72	55	51	47	43	39	0
			UMS	100	80	70	60	50	40	0
4773	01	DC – Decision mathematics computation (A2)	Raw	72	46	40	34	29	24	0
			UMS	100	80	70	60	50	40	0
4776	01	(NM) Numerical Methods (AS): Written Paper	Raw	72	57	52	48	44	39	0
4776	02	(NM) Numerical Methods (AS): Coursework	Raw	18	14	12	10	8	7	0
4776	82	(NM) Numerical Methods (AS): Carried Forward Coursework Mark	Raw	18	14	12	10	8	7	0
			UMS	100	80	70	60	50	40	0
4777	01	NC – Numerical computation (A2)	Raw	72	55	47	39	32	25	0
			UMS	100	80	70	60	50	40	0
4798	01	FPT - Further pure mathematics with technology (A2)	Raw	72	57	49	41	33	26	0
			UMS	100	80	70	60	50	40	0

AS GCE Statistics (MEI)

				Max Mark	a	b	c	d	e	u
G241	01	Statistics 1 MEI	Raw	72		No entry in June 2018				
			UMS	100	80	70	60	50	40	0
G242	01	Statistics 2 MEI	Raw	72		No entry in June 2018				
			UMS	100	80	70	60	50	40	0
G243	01	Statistics 3 MEI	Raw	72		No entry in June 2018				
			UMS	100	80	70	60	50	40	0

AS GCE Quantitative Methods (MEI)

				Max Mark	a	b	c	d	e	u
G244	01	Introduction to Quantitative Methods (Written Paper)	Raw	72	58	50	43	36	28	0
G244	02	Introduction to Quantitative Methods (Coursework)	Raw	18	14	12	10	8	7	0
			UMS	100	80	70	60	50	40	0
G245	01	Statistics 1	Raw	72	61	55	49	43	37	0
			UMS	100	80	70	60	50	40	0
G246	01	Decision Mathematics 1	Raw	72	50	44	38	32	26	0
			UMS	100	80	70	60	50	40	0

Level 3 Certificate, Level 3 Extended Project and FSMQ raw mark grade boundaries June 2018 series

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Level 3 Certificate Mathematics - Quantitative Methods (MEI)

					Max Mark	a	b	c	d	e	u
G244	A	01	Introduction to Quantitative Methods with Coursework (Written Paper)	Raw	72	58	50	43	36	28	0
G244	A	02	Introduction to Quantitative Methods with Coursework (Coursework)	Raw	18	14	12	10	8	7	0
				UMS	100	80	70	60	50	40	0
				Overall	90	72	62	53	44	35	0

Level 3 Certificate Mathematics - Quantitative Reasoning (MEI)

				Max Mark	a	b	c	d	e	u
H866	01	Introduction to quantitative reasoning	Raw	72	56	49	42	35	28	0
H866	02	Critical maths	Raw	60	44	39	34	29	24	0
		*To create the overall boundaries, component 02 is weighted to give marks out of 72	Overall	144	109	96	83	70	57	0

Level 3 Certificate Mathematics - Quantitative Problem Solving (MEI)

				Max Mark	a	b	c	d	e	u
H867	01	Introduction to quantitative reasoning	Raw	72	56	49	42	35	28	0
H867	02	Statistical problem solving	Raw	60	40	36	32	28	24	0
		*To create the overall boundaries, component 02 is weighted to give marks out of 72	Overall	144	104	92	80	69	57	0

Advanced Free Standing Mathematics Qualification (FSMQ)

				Max Mark	a	b	c	d	e	u
6993	01	Additional Mathematics	Raw	100	56	50	44	38	33	0

Intermediate Free Standing Mathematics Qualification (FSMQ)

				Max Mark	a	b	c	d	e	u
6989	01	Foundations of Advanced Mathematics (MEI)	Raw	40	35	30	25	20	16	0